

FIVE-MINUTE OSCILLATION: THEORY

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Literature

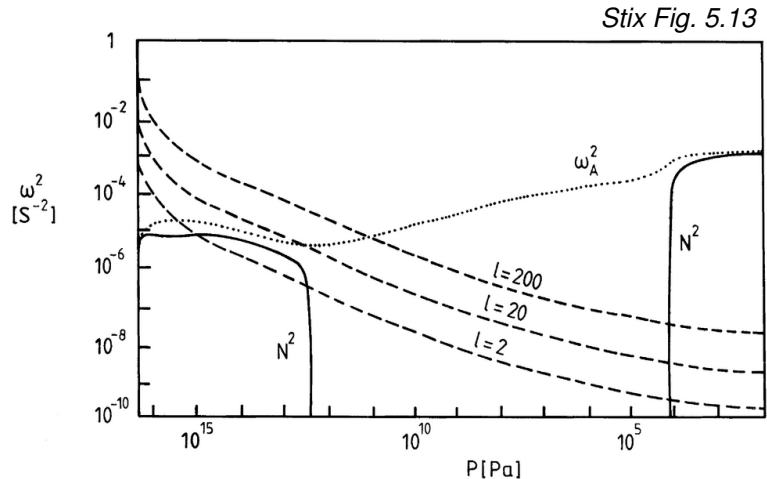
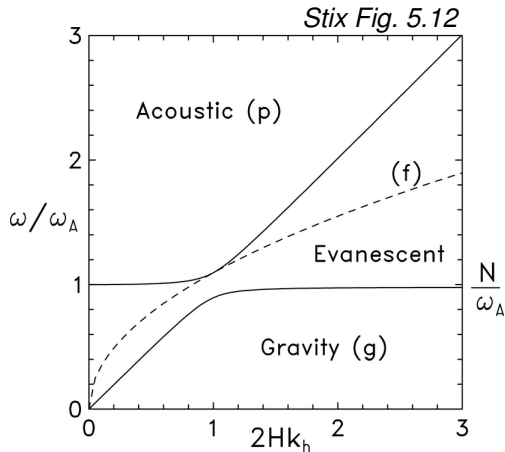
Jørgen Christensen-Dalsgaard: "Stellar Oscillations", 2003

<http://astro.phys.au.dk/~jcd/oscilnotes>

Michael Stix: "The Sun", 2004, second edition, Springer

Rob Rutten: "Fotosferische snelheidsvelden", 1983

http://www.staff.science.uu.nl/~rutte101/Collegedictaat_Fotosferisch.html



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BASIC EQUATIONS

Stix sections 2.3.5, 5.2.1

negligible viscosity, perfect conduction, neglect rotation ($\Omega_{\text{rot}} \approx 27 \text{ days} \approx 10^4 \omega_{\text{osc}}$)
linear perturbations of hydrostatic equilibrium
subsonic velocities $v \ll c_s$ (but **chromospheric shocks** $M \equiv v/c_s \approx 1.1 - 1.5$)

Euler “local” versus Lagrange “material” coordinates

Langrangian property change: δ

$$[\alpha(t+\Delta t)]_{\vec{\delta r}} = \alpha(\vec{r}, t) + \Delta t \left(\left[\frac{\partial \alpha}{\partial t} \right]_{\vec{r}} + \sum_i \frac{\partial \alpha}{\partial x_i} \frac{\partial x_i}{\partial t} \right) \quad \frac{d\alpha}{dt} = \left[\frac{\alpha(t + \Delta t) - \alpha(t)}{\Delta t} \right]_{\vec{\delta r}} = \frac{\partial \alpha}{\partial t} + \vec{v} \cdot \nabla \alpha$$

first law of thermodynamics

$$\frac{dq}{dt} = \frac{dE}{dt} + P \frac{dV}{dt} \quad V = 1/\rho \quad \rho \frac{dq}{dt} = \rho \frac{dE}{dt} - \frac{P}{\rho} \frac{d\rho}{dt}$$

ideal gas

$$\delta E = c_v \delta T \quad P = (c_p - c_v) \rho T \quad P = (\gamma - 1) \rho E \quad \gamma = \frac{c_p}{c_v} \quad \text{ionization: } \Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_{\text{ad}}$$

combine

$$\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt} + (\gamma - 1) \rho \frac{dq}{dt} \quad \text{adiabatic } (\delta q = 0): \frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt}$$

$\delta q \neq 0$: convection (mixing length), radiation (diffusion approx. \Rightarrow Eddington approx. \Rightarrow NLTE)

CONSERVATION EQUATIONS

Stix section 5.2

continuity Euler

$$\int \frac{\partial \rho}{\partial t} dV \equiv \oint -\rho \vec{v} \cdot d\vec{s} \stackrel{\text{Gauss}}{=} - \int \nabla \cdot (\rho \vec{v}) dV \quad \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v}) = -\vec{v} \cdot \nabla \rho - \rho \nabla \cdot \vec{v}$$

continuity Lagrange

$$\frac{d}{dt} (\rho V) = 0 \quad \frac{1}{\rho V} \frac{d}{dt} (\rho V) = \frac{1}{\rho} \frac{d\rho}{dt} + \frac{1}{V} \frac{dV}{dt} = 0 \quad \frac{dV}{dt} = V \nabla \cdot \vec{v} \quad \frac{d\rho}{dt} = -\rho \nabla \cdot \vec{v}$$

momentum

$$\rho \frac{d\vec{v}}{dt} = -\nabla P + \rho \vec{g} + \dots \quad \rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \rho \vec{g} + \dots$$

Lorentz $(\nabla \wedge \vec{B}) \wedge \vec{B}/4\pi$ Coriolis $2\vec{\Omega} \wedge \vec{v}$ differential rotation $(\vec{v} \cdot \nabla \vec{\Omega}) \wedge \vec{v}$ viscosity $\mu \nabla^2 \vec{v}$

Poisson

$$\vec{g} = -\nabla \Phi \quad \nabla^2 \Phi = 4\pi G \rho \quad \Phi(\vec{r}) = -G \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV$$

Energy (adiabatic)

$$\frac{dP}{dt} = \frac{\gamma P}{\rho} \frac{d\rho}{dt}$$

SMALL CHANGES

Stix section 5.2.1

linear perturbations

$$P = P_0 + P_1 \quad \rho = \rho_0 + \rho_1 \quad \vec{v} = \vec{v}_0 + \vec{v}_1 = \vec{v}_1 \quad P_1 \ll P_0 \quad \rho_1 \ll \rho_0 \quad \vec{v} \ll c_s$$

Lagrangian perturbations (S 5.15 with displacement $\vec{\delta r} = \xi$)

$$\delta P = P_1 + \vec{\delta r} \cdot \nabla P_0 \quad \rho = \rho_1 + \vec{\delta r} \cdot \nabla \rho_0 \quad \vec{v} = \frac{\partial \vec{\delta r}}{\partial t}$$

continuity (S 5.13)

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \vec{v}) = 0 \quad \rho_1 + \nabla \cdot (\rho_0 \vec{\delta r}) = 0$$

momentum (S 5.14)

$$\rho_0 \frac{\partial^2 \vec{\delta r}}{\partial t^2} = \rho_0 \frac{\partial \vec{v}}{\partial t} = -\nabla P_1 + \rho_0 \vec{g}_1 + \rho_1 \vec{g}_0 = -\nabla P_1 - \rho_0 \nabla \Phi_1 + \frac{\rho_1}{\rho_0} \nabla P_0$$

Cowling approximation (S 5.2.3, 5.29): waves \Rightarrow many radial sign changes \Rightarrow average out

$$\nabla^2 \Phi_1 = 4\pi G \rho_1 \quad \Phi_1 = -G \int \frac{\rho_1(r')}{|r - r'|} dr' \approx 0$$

adiabatic energy (S 5.10)

$$\frac{P_1}{P_0} = \gamma \frac{\rho_1}{\rho_0} \quad \frac{\delta P}{P_0} = \gamma \frac{\delta \rho}{\rho_0}$$

ACOUSTIC WAVES IN HOMOGENEOUS MEDIUM

momentum, continuity, energy equations without gravity or mean-state derivatives

$$\rho_0 \frac{\partial^2 \delta \vec{r}}{\partial t^2} = -\nabla P_1 \quad \frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \vec{v}_1 \quad P_1 = \gamma \frac{\gamma P_0}{\rho_0} \rho_1$$

with sound speed $c_s^2 \equiv \gamma P_0 / \rho_0$ Mach $M \equiv V / c_s$ ideal gas $c_s^2 = \gamma k T / \mu \sim T / \mu$

$$\frac{\partial^2 P_1}{\partial t^2} = c_s^2 \left(-\rho_0 \nabla \cdot \frac{\partial \vec{v}_1}{\partial t} \right) = c_s^2 \nabla^2 P_1$$

$$\frac{\partial^2 \rho_1}{\partial t^2} = -\rho_0 \nabla \cdot \frac{\partial \vec{v}_1}{\partial t} = c_s^2 \nabla^2 \rho_1$$

$$\frac{\partial^2 \vec{v}_1}{\partial t^2} = -\frac{1}{\rho_0} \nabla \cdot \left(c_s^2 \frac{\partial \rho_1}{\partial t} \right) = c_s^2 \nabla^2 \vec{v}_1$$

space-time variable separation

$$P(x, y, z, t) \equiv g(x, y, z) f(t) \Rightarrow \frac{1}{f} \frac{\partial^2 f}{\partial t^2} = \frac{c_s^2}{g} \nabla^2 g \equiv -\omega^2 \Rightarrow \frac{\partial^2 f}{\partial t^2} + \omega^2 f = 0 \Rightarrow f(t) = a_t e^{i\omega t} + b_t e^{-i\omega t}$$

spatial variable separation $g(x, y, z) = g_x(x) g_y(y) g_z(z)$

$$\nabla^2 g + \frac{\omega^2}{c_s^2} g = 0 \Rightarrow \frac{\partial^2 g_x}{\partial x^2} + k_x^2 g_x = 0 \Rightarrow g_x(x) = a_x e^{ik_x x} + b_x e^{-ik_x x} \text{ etc.}$$

plane waves

$$P_1 = P_0 \gamma M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)} \quad \rho_1 = \rho_0 M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)} \quad \vec{v}_1 = c_s M e^{i(\vec{k} \cdot \vec{r} \pm i\omega t)}$$

spatial wavenumber $\vec{k} = (k_x, k_y, k_z) \parallel \vec{v}$

dispersion relation $k^2 = |\vec{k}|^2 = \omega^2 / c_s^2$

PLANE WAVE PROPERTIES

waves in homogeneous gaseous medium

$$\frac{P_1}{P_0 \gamma M} = \frac{\rho_1}{\rho_0 M} = \frac{\vec{v}_1}{c_s M} = e^{i(\vec{k} \cdot \vec{r} - \omega t)} \quad \vec{k} \parallel \vec{v}_1 \parallel \delta \vec{r} \quad \text{longitudinal}$$

wavelength, wavenumber, period, (angular) frequency

$$\begin{aligned} \text{full cycle at } kx = 2\pi &\Rightarrow x = \lambda & k_x &= 2\pi/\lambda \\ \text{full cycle at } \omega t = 2\pi &\Rightarrow t = P & \omega &= 2\pi/P = 2\pi f \end{aligned}$$

plane wave: same phase across plane with $\vec{k} \cdot \vec{r} = 0$ so $\perp \vec{k}$ and $\perp \vec{v}$

phase velocity = propagation wave pattern

$$\text{wave } \vec{v} = (\dot{x}, 0, 0): \text{ equal phase: } k_x x - \omega t = c \quad x = (\omega/k_x) t + c \quad v_{\text{phase}} = \omega/k = c_s$$

group velocity = propagation envelope non-monochromatic wave train $L_x \Delta k_x \approx 1/2$

$$e^{i[(\vec{k} + \Delta \vec{k}) \cdot \vec{r} - (\omega + \Delta \omega)t]} = e^{i(\vec{k} \cdot \vec{r} - \omega t)} e^{i(\Delta \vec{k} \cdot \vec{r} - \Delta \omega t)} \Rightarrow v_{\text{group}} = \frac{\partial \omega}{\partial k}$$

dispersion relation $|\vec{k}|^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2/c_s^2$ and diagnostic diagram

shaded: real solutions = propagating waves

boundary line: $k_y = k_z = 0$, waves only in x

smaller k_x towards upper left

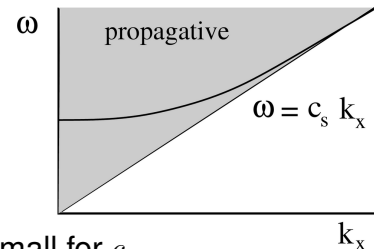
ω axis: $k_x = 0$, waves only in y and/or z

curve: $\omega = \sqrt{c_s^2 k_x^2 + \bar{c}}$ for constant $k_y^2 + k_z^2 = \bar{c}$

long periods: only at large wavelength $\lambda_x = 2\pi/k_x$

non-shaded: period P too long for k_x , wavelength λ_x too small for c_s

imaginary solutions $k_y^2 < 0$ or $k_z^2 < 0$: exponential growth/decay in y or z



VERTICAL ACOUSTIC WAVES IN ISOTHERMAL ATMOSPHERE

“atmosphere” \equiv gas layer plane-parallel in x and y , stratified by gravity in z

$$\nabla T_0 = 0 \quad \nabla P_0 = \rho_0 \vec{g} \Rightarrow \frac{\partial P_0}{\partial z} = -\rho_0 g = -P_0 \frac{\gamma g}{c_s^2} \equiv -\frac{P_0}{H} \quad H = \frac{c_s^2}{\gamma g} = \frac{RT}{\mu g} \quad k_h^2 \equiv k_x^2 + k_y^2$$

linearized conservation equations with $\vec{v} = (v_h, v_z)$ and $\vec{g} = (0, g)$

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot \vec{v} - \vec{v} \cdot \nabla \rho_0 \quad \rho_0 \frac{\partial \vec{v}}{\partial t} = \rho_1 \vec{g} - \nabla P_1 \quad \frac{\partial P_1}{\partial t} + \vec{v} \cdot \nabla P_0 = -\gamma P_0 \nabla \cdot \vec{v}$$

give inhomogeneous wave equations for P_1, ρ_1, \vec{v}

$$\frac{\partial^2 \vec{v}}{\partial t^2} = c_s^2 \nabla^2 \vec{v} - (\gamma - 1) \vec{g} \nabla \cdot \vec{v} - \nabla(\vec{v} \cdot \vec{g})$$

vertical motion $\vec{v} = (0, v_z)$

$$\frac{\partial^2 v_z}{\partial t^2} = c_s^2 \nabla^2 v_z - \gamma g \frac{\partial v_z}{\partial z} \quad \text{try } v_z = e^{i(k_z z - \omega t)}$$

$$\omega^2 = c_s^2 k_z^2 + i \gamma g k_z \quad k_z = -i \frac{\gamma g}{2c_s^2} \pm \frac{1}{c_s} \sqrt{\omega^2 - \omega_a^2} \quad \omega_a \equiv \frac{\gamma g}{2c_s}$$

$$v_z = e^{(\gamma g / 2c_s^2) z} e^{i \left[\pm (z/c_s) \sqrt{\omega^2 - \omega_a^2} - \omega t \right]} = e^{z/2H} e^{i \left[\pm (z/c_s) \sqrt{\omega^2 - \omega_a^2} - \omega t \right]}$$

properties

amplitude $\sim e^{z/2H} \sim \sqrt{\rho_0(z)} =$ energy conservation $(1/2)\rho_0 v_z^2$ up to non-linear regime

$\omega > \omega_a$ propagating plane wave

$\omega < \omega_a$ slow perturbation = evanescent wave: whole atmosphere up and down in phase
aperiodic growth or decay with z depending on upper or lower piston

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 1

Stix section 5.2.4

Hines 1960CaJPh..38.1441H

Whitaker 1963ApJ...137..914W

substitute into linearised conservation laws

$$\frac{P_1}{P_0 \mathcal{P}} = \frac{\rho_1}{\rho_0 \mathcal{R}} = \frac{v_h}{\mathcal{X}} = \frac{v_z}{\mathcal{Z}} = e^{i(k_h \cdot x + k_z \cdot z - \omega t)} \quad \text{complex } \mathcal{P}, \mathcal{R}, \mathcal{X}, \mathcal{Z}, k_h, k_z$$

separation of vertical-only solution: $\Im k_z = \gamma g / 2c_s^2$

$$\frac{P_1}{P_0 \mathcal{P}} = \frac{\rho_1}{\rho_0 \mathcal{R}} = \frac{v_h}{\mathcal{X}} = \frac{v_z}{\mathcal{Z}} = e^{z/2H} e^{i(k_h x + k_z z - \omega t)} \quad \text{real } k_h, k_z$$

polarisation relations

$$\begin{aligned} \mathcal{P} &= \gamma \omega^2 \left[k_z - i \left(1 - \frac{\gamma}{2}\right) \frac{g}{c_s^2} \right] & \mathcal{R} &= \omega^2 k_z + i(\gamma - 1) g k_h^2 - i \frac{\gamma}{2} \frac{g \omega^2}{c_s^2} \\ \mathcal{X} &= \omega k_h c_s^2 \left[k_z - i \left(1 - \frac{\gamma}{2}\right) \frac{g}{c_s^2} \right] & \mathcal{Z} &= \omega (\omega^2 - k_h^2 c_s^2) \end{aligned}$$

amplitude & phase

$$\mathcal{X} \equiv |\mathcal{X}| e^{i\phi} \quad v_h = |\mathcal{X}| e^{z/2H} e^{i(k_h x + k_z z - \omega t + \phi)} \quad \phi = \arctan(\Im \mathcal{X} / \Re \mathcal{X})$$

dispersion relation

$$\omega^4 - \omega^2 c_s^2 (k_h^2 + k_z^2) + (\gamma - 1) g^2 k_h^2 - \frac{\gamma^2 g^2 \omega^2}{4c_s^2} = 0$$

$$\omega_a \equiv \frac{\gamma g}{2c_s} \quad \omega_g \equiv \frac{g}{c_s} \sqrt{\gamma - 1} \quad \omega_g < \omega_a \text{ since } \gamma < 2 \quad \gamma = 5/3 \Rightarrow \omega_g = 0.98 \omega_a$$

$$(\omega^2 - \omega_a^2) \frac{\omega^2}{c_s^2} - \omega^2 (k_h^2 + k_z^2) + \omega_g^2 k_h^2 = 0$$

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 2

reordered dispersion relations

$$k_h^2 = \frac{(\omega_a^2 - \omega^2)(\omega^2/c_s^2) + \omega^2 k_z^2}{\omega_g^2 - \omega^2} \quad k_z^2 = (\omega_g^2 - \omega^2) \frac{k_h^2}{\omega^2} - (\omega_a^2 - \omega^2)/c_s^2$$

$$\omega^2 = \frac{1}{2} \left[\omega_a^2 + c_s^2 k^2 \pm \sqrt{(\omega_a^2 + c_s^2 k^2)^2 - 4 c_s^2 k_h^2 \omega_g^2} \right]$$

Brunt-Väisälä frequency Schwarzschild convective instability criterion: $N^2 < 0$

$$\omega_g \equiv \frac{g}{c_s} \sqrt{\gamma - 1} \quad N_{\text{BV}}^2 \equiv \frac{g^2}{c_s^2} (\gamma - 1) + \frac{g}{T} \frac{dT}{dz} \quad \text{isothermal} \quad \omega_g = N_{\text{BV}} \quad \gamma = 5/3 : \omega_g = 0.98 \omega_a$$

with $L^2 \equiv c_s^2 k_h^2$ and $\omega_g \approx \omega_a$

$$k_z^2 = \frac{1}{\omega^2 c_s^2} [(\omega_g^2 - \omega^2) k_h^2 c_s^2 - (\omega_a^2 - \omega^2) \omega^2] \approx \frac{1}{\omega^2 c_s^2} (\omega^2 - \omega_{g \approx a}^2) (\omega^2 - L^2)$$

diagnostic diagram

shaded: two ω roots per (k_h, k_z) = propagative waves

$\omega > \omega_a$ and $\omega > L$: acoustic wave

$\omega < \omega_g$ and $\omega < L$: internal gravity wave

Cowling: p (pressure) or g (gravity)

blank: $k_z^2 < 0$: evanescent in z

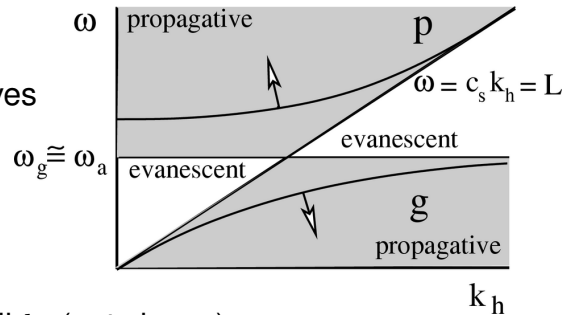
line $\omega^2 = c_s^2 k_h^2 = L^2$: horizontal “Lamb” waves

line $\omega^2 = (\omega_g/\omega_a)^2 c_s^2 k_h^2$: gravity-wave cutoff at small k_h (not shown)

upper curve: constant k_z , hyperbolic, $\omega^2 - \omega_a^2 \approx c_s^2 k_z^2$ for small k_h

lower curve: constant k_z , hyperbolic, $\omega_g^2 - \omega^2 \approx k_z^2/k_h^2$ for large k_h

arrows: increasing k_z



SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 3

phase speed $\theta \equiv$ angle between \vec{k} and k_z

$$v_{\text{phase}}^2 = \frac{\omega^2}{k^2} = \frac{1}{2} \left[\frac{\omega_a^2}{k^2} + c_s^2 \pm \sqrt{\left(\frac{\omega_a^2}{k^2} + c_s^2 \right)^2 - 4 c_s^2 \sin^2 \theta \frac{\omega_g^2}{k^2}} \right] \omega / \omega_A$$

$+\sqrt{\dots} = p$ waves, $-\sqrt{\dots} = g$ waves

p waves: $v_{\text{phase}} \geq c_s$

$v_{\text{phase}} \downarrow$ for $\omega \uparrow$

g waves: $0 \leq v_{\text{phase}} \leq (\omega_g / \omega_a) c_s$

$v_{\text{phase}} \uparrow$ for $\theta \rightarrow \pi/2$

$v_{\text{phase}} = 0$ for $\theta = 0, \pi$

group velocity

$$v_{\text{group}}^{\text{hor}} = \frac{c_s^2 (\omega^2 - \omega_g^2) k_h}{\omega (2\omega^2 - \omega_a^2 - c_s^2 k^2)}$$

$$v_{\text{group}}^{\text{ver}} = \frac{c_s^2 \omega^2 k_z}{\omega (2\omega^2 - \omega_a^2 - c_s^2 k^2)}$$

pure p regime $\omega \gg \omega_a$

$$\omega^2 = c_s^2 k^2 \quad v_{\text{phase}} \approx c_s \quad v_{\text{group}}^{\text{hor,ver}} \approx (c_s^2 / \omega) k_{h,v} \leq c_s \quad P_1, \rho_1, v_h, v_z \text{ in phase}$$

pure g regime $k_h^2 \gg \omega_g^2 / c_s^2$

$$\text{dispersion: } \omega^2 = (k_h^2 / k^2) \omega_g^2 = \sin^2 \theta \omega_g^2$$

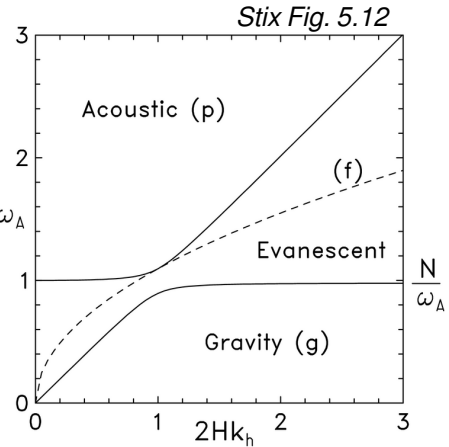
non-vertical only ("blob" concept needs horizontal info)

$v_{\text{group}}^{\text{hor}} \sim k_h, v_{\text{group}}^{\text{ver}} \sim -k_z$ slanted upward waves transport energy downward

evanescent regime

k_z imaginary, k_h free P, \mathcal{R} imaginary, \mathcal{Z} real: P_1 and \vec{v}_z 90 deg out of phase

non-radial fundamental mode (f) \sim ocean wind wave: $\omega = \sqrt{g k_h}$



WAVE TRAPPING

Stein & Leibacher 1974ARA&A..12..407S

temperature sensitivities

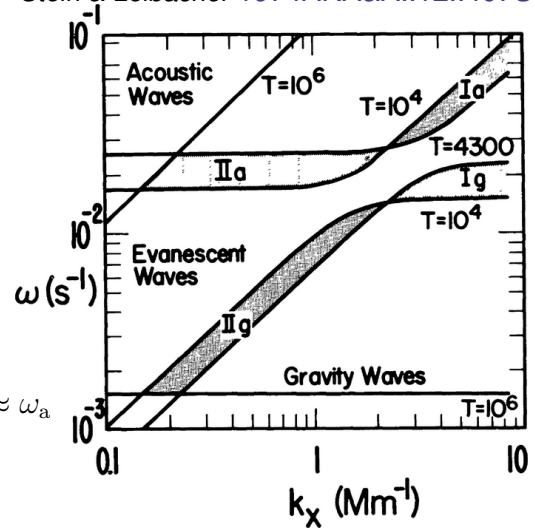
$$\text{sound speed: } c_s \equiv \sqrt{\gamma \frac{P_0}{\rho_0}} = \sqrt{\gamma \frac{RT}{\mu}} \sim \sqrt{T}$$

$$\text{acoustic cutoff frequency: } \omega = \omega_a \equiv \frac{\gamma g}{2 c_s} \sim 1/\sqrt{T}$$

$$\text{horizontal acoustic-wave line (Lamb): } \omega = c_s k_h$$

$$\text{gravity cutoff line: } \omega = (\omega_g/\omega_a) c_s k_h$$

$$\text{horizontal gravity-wave frequency: } \omega = N_{BV} \approx \omega_g \approx \omega_a$$



wave trapping

refraction at increasing wave speed (wavefront bending)

turn-around where waves become horizontal (node in k_z)

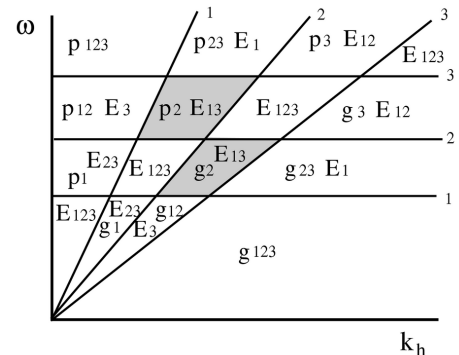
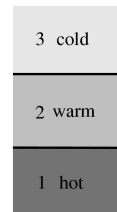
anti-node reflection at cutoff (infinite phase speed)

p -mode cavity walls: ω_a cutoff and Lamb-line refraction

g -mode cavity walls: gravity cutoff line and N_{BV} refraction

example: three temperature regimes

shaded: 2 = cavity p and g modes



SOLAR CAVITIES

p : refractive turnaround at Lamb line

$$\omega^2 = c_s^2 k_h^2 = c_s^2 \frac{l(l+1)}{r^2} \sim \frac{T l^2}{r^2}$$

lowest l modes probe deepest

g : refractive turnaround at Brunt-Väisälä N^2

no g waves in convection zone

interior g modes? lift whole envelope?

atmospheric g waves?

near solar center

Lamb reflection $\omega^2 \sim 1/r^2$

ω_a and $N_{BV} \sim g \sim r \Rightarrow N^2 \sim r^2$

only radial $p(l=0)$ modes reach center

five-minute oscillation ($\omega \approx 0.02$ Hz)

upper reflection = ω_a (here N_{AC})

interior reflection = Lamb turnaround
(\approx closed/open ends of my flute)

p modes evanescent where detected

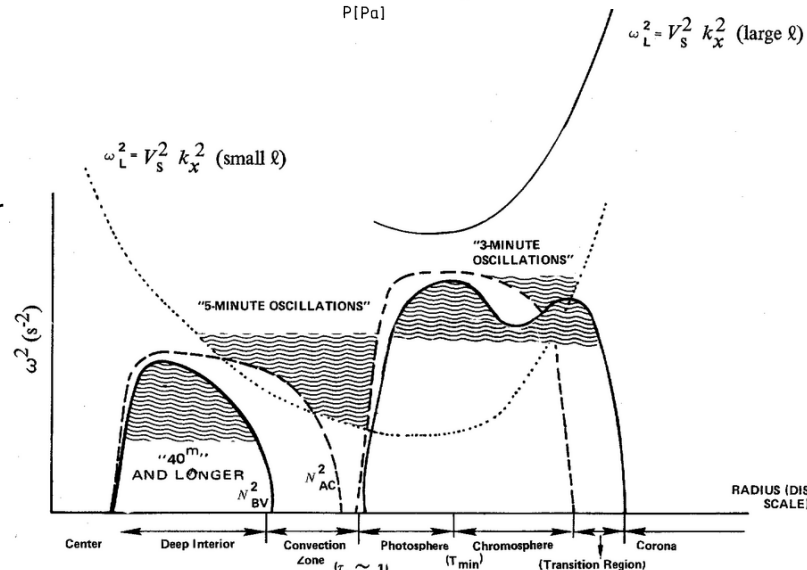
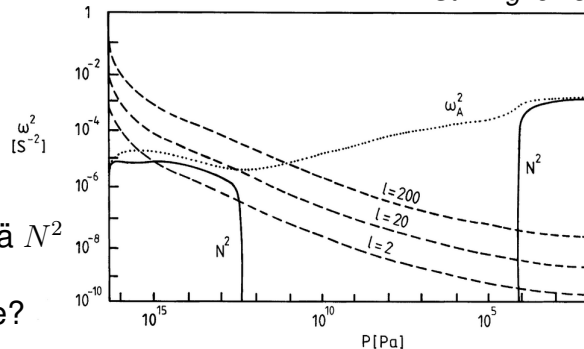
three-minute oscillation ($\omega \approx 0.035$ Hz)

here: cavity from coronal T rise

but: T rise too warped

reflection/mode conversion at \vec{B}

Stix Fig. 5.13



Leibacher & Stein 1981 [suas.nasa..263L](https://doi.org/10.26434/chemrxiv-2019-08)

NUMERICAL p -MODE PREDICTION

Ando & Osaki 1975PASJ...27..581A

(a) *Basic Equations and Boundary Conditions*

The basic equations governing nonadiabatic radial pulsations in the radiative atmosphere were formulated by UNNO (1965), in which the radiative transfer was treated in the Eddington approximation. UNNO and SPIEGEL (1966) have demonstrated that the Eddington approximation in the radiative heat equation is very useful in three-dimensional time-dependent problems. We thus utilize this in the present paper. The basic equations governing nonadiabatic nonradial oscillations in the radiative atmosphere are then given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla P - \mathbf{g}, \quad (2)$$

$$c_P \rho \left(\frac{dT}{dt} - \nabla_{\text{ad}} \frac{T}{P} \frac{dP}{dt} \right) = -\pi \nabla \cdot \mathbf{F}, \quad (3)$$

$$\mathbf{F} = -\frac{4}{3\kappa\rho} \nabla J, \quad (4)$$

and

$$J = \frac{ac}{4\pi} T^4 + \frac{c_P}{4\pi\kappa} \left(\frac{dT}{dt} - \nabla_{\text{ad}} \frac{T}{P} \frac{dP}{dt} \right), \quad (5)$$

THE SUN IS A SPHERE

Wolff 1972ApJ...177L..87W

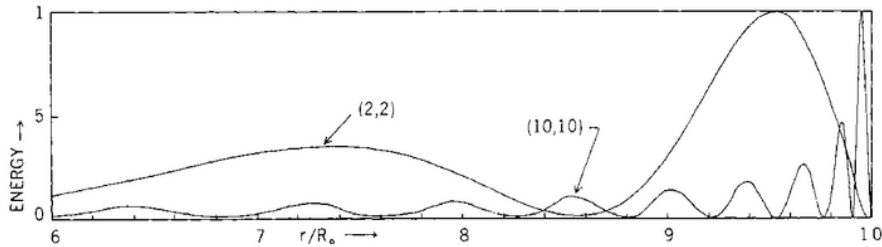
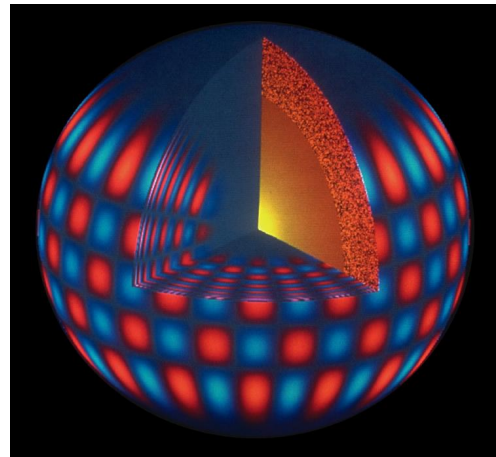
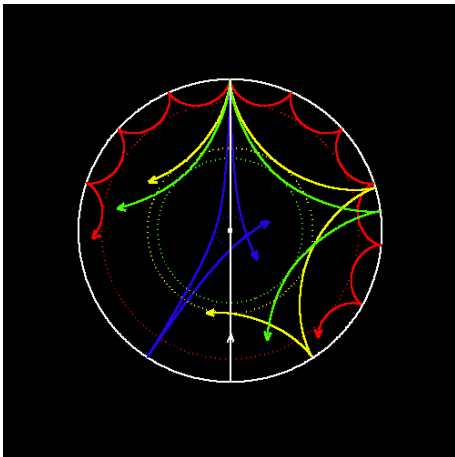


FIG. 1.—Radial distribution of kinetic energy (arbitrary units) in spherical shells of unit thickness for two nonradial p -modes of solar oscillation.



NUMERICAL p -MODE PREDICTION

Ando & Osaki 1975PASJ...27..581A

We assume as usual that the small perturbation of a physical quantity f is written as

$$\left. \begin{array}{l} f'(r, \theta, \varphi, t) \\ \delta f(r, \theta, \varphi, t) \end{array} \right\} = \left. \begin{array}{l} f'(r) \\ \delta f(r) \end{array} \right\} Y_l^m(\theta, \varphi) e^{i\sigma t}, \quad (6)$$

where (r, θ, φ) is the spherical polar coordinates, the Eulerian and the Lagrangian perturbations are denoted by prime ($'$) and δ , and $Y_l^m(\theta, \varphi)$ is the spherical harmonics. We introduce a nondimensional frequency ω and five nondimensional variables x, p, θ, j , and λ defined by

$$\left. \begin{array}{l} \omega^2 = (R^3/GM)\sigma^2, \\ x = \delta r/r, \quad p = P'/\rho g r, \quad \theta = \delta T/T, \\ j = \delta J/J, \quad \lambda = \delta L_r/L_s, \end{array} \right\} \quad (7)$$

where $L_r = 4\pi^2 r^2 F$ is the luminosity at radius r and L_s is the luminosity at the surface. By linearizing equations (1)-(5), we then obtain four first-order linear differential equations and one auxiliary equation:

POWER RIDGES $f-l$ (aka $k-\omega$) DIAGRAM

Stix section 5.2

assume constant temperature gradient with depth $d = -z$

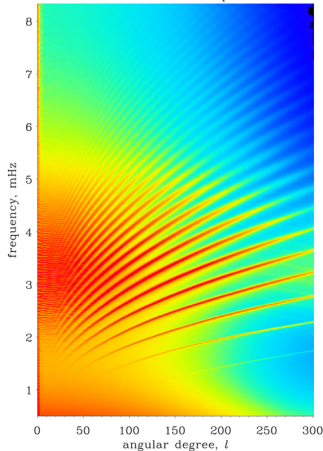
$$\frac{dT}{dd} \approx C = \left(\frac{dT}{dd}\right)_{\text{ad}} \quad T(d) = \left(\frac{dT}{dd}\right)_{\text{ad}} d \quad -\frac{1}{T} \left(\frac{dT}{dd}\right)_{\text{ad}} = \frac{(\gamma-1)g}{c_s^2} \quad c_s^2 \approx (\gamma-1)gd$$

lower reflection occurs at depth $d = \delta$ where $\omega^2 = c_s^2 k_h^2$: $\delta = \frac{c_s^2}{(\gamma-1)g} = \frac{\omega^2}{(\gamma-1)g k_h^2}$

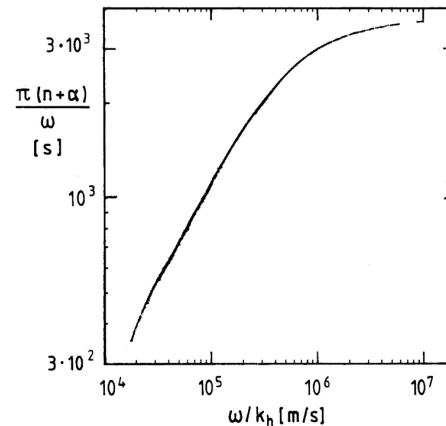
standing wave has $(n+1/2)\pi = \int_{d=\delta}^{d=0} k_z dd = \omega \int_{d=\delta}^{d=0} \frac{1}{c_s} dd = \frac{2\omega\sqrt{\delta}}{\sqrt{(\gamma-1)g}} = \frac{2\omega^2}{(\gamma-1)g k_h}$

parabolas $\omega_n^2 = \frac{1}{2}\pi(n+1/2)(\gamma-1)g k_h$ $k_h^2 = l(l+1)/R_\odot^2$

<http://sohowww.nascom.nasa.gov>



Stix Fig. 5.16



Duvall's law with $\alpha = 1.58$

FIVE-MINUTE OSCILLATION: THEORY

Robert J. Rutten

<http://www.staff.science.uu.nl/~rutte101>

Literature

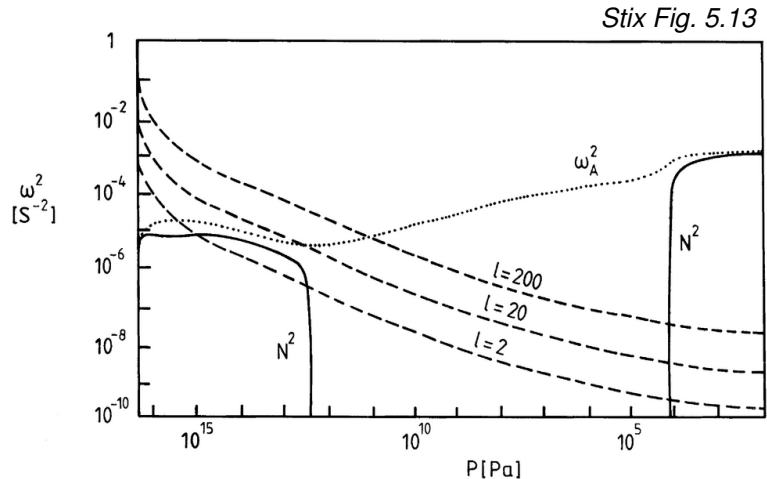
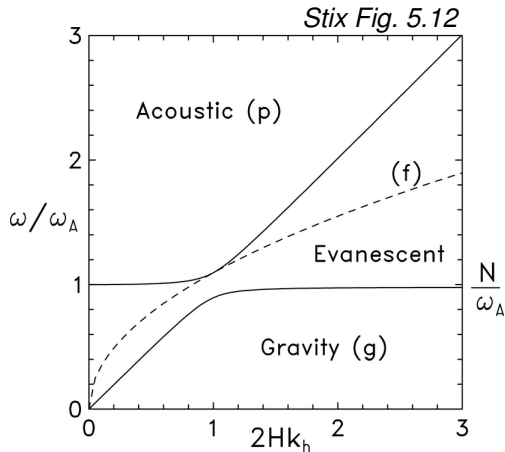
Jørgen Christensen-Dalsgaard: "Stellar Oscillations", 2003

<http://astro.phys.au.dk/~jcd/oscilnotes>

Michael Stix: "The Sun", 2004, second edition, Springer

Rob Rutten: "Fotosferische snelheidsvelden", 1983

http://www.staff.science.uu.nl/~rutte101/Collegedictaat_Fotosferisch.html



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FIVE-MINUTE OSCILLATION THEORY

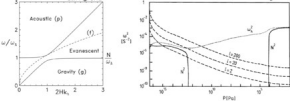
Robert J. Rutten

https://www.staff.science.uu.nl/~rutte101

Literature
 Jørgen Christensen-Dalsgaard: "Solar Oscillations", 2003
 http://astro.phys.uu.dk/~jcd/oscillations

Michael Stix: "The Sun", 2004, second edition, Springer
 Rob Rutten: "Voskresenie smeshchivatel'nykh", 1993
 http://www.staff.science.uu.nl/~rutte101/Colligedictaat...
 fscoteclab.html

Stix Fig. 5.12



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1

SMALL CHANGES

Stix section 5.2.1

linear perturbations
 $P = P_0 + \delta P$ $\rho = \rho_0 + \delta \rho$ $v = v_0 + \delta v$ $\tilde{P}_1 \ll \tilde{P}_0$ $\tilde{\rho}_1 \ll \tilde{\rho}_0$ $\tilde{v}_1 \ll \tilde{v}_0$

Lagrangian perturbations (S.5.15) with displacement $\tilde{r} = \xi$
 $\delta P = \tilde{P}_1 + \delta \rho \tilde{V}_1$ $\rho = \rho_0 + \delta \rho$ $\nabla \rho = \nabla \rho_0 + \delta \rho \nabla \rho_0$ $\tilde{v} = \frac{d\tilde{r}}{dt}$

continuity (S.5.13)
 $\frac{d\tilde{\rho}_1}{dt} + \tilde{\rho}_1 \nabla \cdot \tilde{v} = 0$ $\rho_0 + \nabla \cdot (\rho_0 \tilde{v}) = 0$

momentum (S.5.14)
 $\rho_0 \frac{d\tilde{v}}{dt} = -\nabla \tilde{P}_1 + \rho_0 \tilde{g} + \rho_0 \tilde{g} = -\nabla \tilde{P}_1 + \rho_0 \nabla \phi + \frac{d}{dt} \nabla \tilde{r}_1$

Cowling approximation (S.5.2, S.29): waves = many radial sign changes = average out
 $\nabla \phi = \text{tr}(G_0)$ $\tilde{\phi}_1 = -G \int \frac{\tilde{\rho}_1}{\rho_0} dr$ $\tilde{g}' = 0$

adiabatic energy (S.5.10)
 $\tilde{P}_1 = \tilde{P}_1$ $\tilde{\rho}_1 = \tilde{\rho}_1$ $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{\rho}_1}{dt}$

4

VERTICAL ACoustic WAVES IN ISOTHERMAL ATMOSPHERE

"atmosphere" = gas layer plane-parallel in x and y stratified by gravity in z
 $\nabla T = 0$ $\nabla P_0 = \rho_0 \tilde{g} = \frac{dP_0}{dz} = -\rho_0 \tilde{g} = -P_0 \frac{d}{dz} \ln \rho_0$ $\tilde{P} = \frac{d}{dz} \frac{d\tilde{r}}{dt}$ $\tilde{P}^2 = \tilde{P}_1^2 + \tilde{P}_2^2$

linearized conservation equations with $\tilde{r} = (r_1, r_2)$ and $\tilde{v} = (v_1, v_2)$
 $\frac{d\tilde{\rho}_1}{dt} = -\rho_0 \nabla \cdot \tilde{v} - \tilde{\rho}_1 \nabla \cdot v_0$ $\rho_0 \frac{d\tilde{v}_1}{dt} = \rho_0 \tilde{v}_1 \nabla \cdot v_0 - \nabla P_0 \tilde{v}_1$ $\frac{d\tilde{r}_1}{dt} + \tilde{r}_1 \nabla \cdot v_0 = -\tilde{P}_1 \nabla \cdot v_0$

give homogeneous wave equations for $\tilde{P}_1, \tilde{r}_1, \tilde{v}_1$
 $\frac{d^2 \tilde{P}_1}{dt^2} = c^2 \nabla^2 \tilde{P}_1 - (\gamma - 1) \tilde{v}_1 \nabla \cdot \nabla P_0$

vertical motion $\tilde{r} = (r_1, r_2)$
 $\rho_0 \frac{d^2 \tilde{r}_1}{dt^2} = -\tilde{P}_1 \nabla \cdot v_0 - \rho_0 \tilde{g} \tilde{r}_1$ $\text{By } v_0 = v_0(z, t - \tau)$

$\tilde{v}^2 = \tilde{v}_1^2 + \tilde{v}_2^2$ $\tilde{v}_1^2 = \tilde{v}_2^2$ $\tilde{v}_1 = \tilde{v}_2$ $\tilde{v}_1 = \frac{\tilde{P}_1}{\rho_0 c}$

$c_0 = \sqrt{\gamma P_0 / \rho_0} = \sqrt{\gamma (1/\rho_0) \frac{dP_0}{dz} / \rho_0} = \sqrt{\gamma \frac{d \ln P_0}{d \ln \rho_0}}$

properties
 amplitude $\sim e^{-\alpha z} \sim \sqrt{\rho_0(z)}$ = energy conservation $(1/2)\rho_0 v^2$ up to non-linear regime
 $\omega > \omega_c$ propagating plane wave
 $\omega < \omega_c$ slow perturbation = evanescent wave: whole atmosphere up and down in phase
 apocyclic growth or decay with z : depending on upper or lower piston

7

BASIC EQUATIONS

Stix sections 2.3.5, 5.2.1

negligible viscosity, perfect conduction, neglect rotation ($\tau_{\text{rot}} = 27$ days = $10^6 \omega^{-1}$)
 linear perturbations of hydrostatic equilibrium
 subsonic velocities $v \ll c_s$ (but chromospheric shocks $M = v/c_s \approx 1.1 - 1.3$)

Euler "local" versus Lagrange "material" coordinates Langrangian property change: δ
 $[\delta(r) + \Delta r]_L = \delta(r) + \Delta r \left(\frac{dr}{dr} \right) + \sum \frac{\partial \delta r_i}{\partial r_j} \frac{dr_j}{dr} = \frac{d[\delta(r) + \Delta r]}{dr} = \frac{d\delta r}{dr} + \frac{d\Delta r}{dr} = \frac{d\delta r}{dr} + \frac{d\Delta r}{dr}$

first law of thermodynamics
 $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \frac{dP}{dr} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$ $\tilde{v} = \tilde{v}$ $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$ $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$

ideal gas
 $\delta E = c_v \delta T$ $P = (\gamma - 1) \rho E$ $\gamma = \frac{c_p}{c_v}$ ionization: $\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_T$

combine
 $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$ $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$ $\frac{dP}{dt} = \frac{dP}{dt} + \tilde{v} \cdot \nabla P$

$k_z \neq 0$ convection (mixing length, radiation diffusion approx. = Eddington approx. = NLTE)

2

ACOUSTIC WAVES IN HOMOGENEOUS MEDIUM

momentum, continuity, energy equations without gravity or mean-state derivatives
 $\rho_0 \frac{d\tilde{v}}{dt} = -\nabla \tilde{P}_1$ $\frac{d\tilde{\rho}_1}{dt} = -\rho_0 \nabla \cdot \tilde{v}$ $\tilde{P}_1 = \gamma \tilde{P}_0$ $\tilde{\rho}_1 = \gamma \tilde{P}_0$

with sound speed $c^2 = \gamma P_0 / \rho_0$ Mach $M = |v|/c$ ideal gas $c^2 = \gamma P_0 / \rho_0$
 $\frac{d\tilde{P}_1}{dt} = -\tilde{v} \cdot \nabla P_0$ $\frac{d\tilde{P}_1}{dt} = -\tilde{v} \cdot \nabla P_0$ $\frac{d\tilde{P}_1}{dt} = -\tilde{v} \cdot \nabla P_0$

space-time variable interaction
 $\tilde{P}_1(x, z, t) = \int \tilde{P}_1(x, z, t') f(t') dt'$ $\tilde{v}_1(x, z, t) = \int \tilde{v}_1(x, z, t') f(t') dt'$ $\tilde{P}_1(x, z, t) = \int \tilde{P}_1(x, z, t') f(t') dt'$

spatial variable interaction $\tilde{P}_1(x, z, t) = \int \tilde{P}_1(x, z, t') f(z') dz'$
 $\tilde{v}_1(x, z, t) = \int \tilde{v}_1(x, z, t') f(z') dz'$ $\tilde{P}_1(x, z, t) = \int \tilde{P}_1(x, z, t') f(z') dz'$

plane waves
 $\tilde{P}_1 = \tilde{P}_1 e^{i(kx - \omega t)}$ $\tilde{v}_1 = \tilde{v}_1 e^{i(kx - \omega t)}$ $\tilde{P}_1 = \tilde{P}_1 e^{i(kx - \omega t)}$ $\tilde{v}_1 = \tilde{v}_1 e^{i(kx - \omega t)}$

total wavenumber $\tilde{K} = (k_x, k_y, k_z)$ dispersion relation $\tilde{K}^2 = \tilde{\omega}^2 / c^2$

5

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 1

Stix section 5.2.4 Hines 1930/31Mn. 38.1441H Whitaker 1933ApJ. 137. 814W

substitute into linearized conservation laws
 $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla P_0$ $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla P_0$ $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla P_0$

separation of vertical only solution: $\tilde{P}_1(x, z, t) = \tilde{P}_1(x, z, t) f(z)$ real k_x, k_y

polarization relations
 $\tilde{P}_1 = \tilde{P}_1 e^{i(k_x x + k_y y - \omega t)}$ $\tilde{v}_1 = \tilde{v}_1 e^{i(k_x x + k_y y - \omega t)}$ $\tilde{P}_1 = \tilde{P}_1 e^{i(k_x x + k_y y - \omega t)}$

amplitude & phase
 $\tilde{P}_1 = |\tilde{P}_1| e^{i\phi}$ $\tilde{v}_1 = |\tilde{v}_1| e^{i\phi}$ $\tilde{P}_1 = |\tilde{P}_1| e^{i\phi}$ $\tilde{v}_1 = |\tilde{v}_1| e^{i\phi}$

dispersion relation
 $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$

8

CONSERVATION EQUATIONS

Stix section 5.2

continuity Euler
 $\frac{d\tilde{\rho}_1}{dt} = \frac{d\tilde{\rho}_1}{dt} + \tilde{v} \cdot \nabla \tilde{\rho}_1 = \frac{d\tilde{\rho}_1}{dt} + \tilde{v} \cdot \nabla \tilde{\rho}_1$ $\frac{d\tilde{\rho}_1}{dt} = \frac{d\tilde{\rho}_1}{dt} + \tilde{v} \cdot \nabla \tilde{\rho}_1$ $\frac{d\tilde{\rho}_1}{dt} = \frac{d\tilde{\rho}_1}{dt} + \tilde{v} \cdot \nabla \tilde{\rho}_1$

continuity Lagrange
 $\frac{d\tilde{\rho}_1}{dt} = 0$ $\frac{d\tilde{\rho}_1}{dt} = 0$ $\frac{d\tilde{\rho}_1}{dt} = 0$ $\frac{d\tilde{\rho}_1}{dt} = 0$

momentum
 $\rho_0 \frac{d\tilde{v}}{dt} = -\nabla \tilde{P}_1 + \rho_0 \tilde{g}$ $\rho_0 \frac{d\tilde{v}}{dt} = -\nabla \tilde{P}_1 + \rho_0 \tilde{g}$ $\rho_0 \frac{d\tilde{v}}{dt} = -\nabla \tilde{P}_1 + \rho_0 \tilde{g}$

Lorentz $(\nabla \cdot \tilde{v}) / \tilde{v}$ Coriolis $2\tilde{v} \times \tilde{\omega}$ differential rotation $(\tilde{v} \cdot \nabla) \tilde{v}$ viscosity $\nabla^2 \tilde{v}$

Poisson
 $\tilde{P}_1 = -\tilde{v} \cdot \nabla P_0$ $\tilde{v} \cdot \nabla = \tilde{v} \cdot \nabla$ $\tilde{P}_1 = -\tilde{v} \cdot \nabla P_0$

Energy (adiabatic)
 $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla \tilde{P}_1$ $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla \tilde{P}_1$ $\frac{d\tilde{P}_1}{dt} = \frac{d\tilde{P}_1}{dt} + \tilde{v} \cdot \nabla \tilde{P}_1$

3

PLANE WAVE PROPERTIES

waves in homogeneous gaseous medium
 $\tilde{P}_1 = \tilde{P}_1 e^{i(kx - \omega t)}$ $\tilde{v}_1 = \tilde{v}_1 e^{i(kx - \omega t)}$ $\tilde{P}_1 = \tilde{P}_1 e^{i(kx - \omega t)}$

wavelength, wavenumber, period, (angular) frequency
 full cycle at $kx = 2\pi \Rightarrow x = \lambda$ $k_x = 2\pi/\lambda$
 full cycle at $\omega t = 2\pi \Rightarrow t = T$ $\omega = 2\pi/T$

plane wave: same phase across plane with $\tilde{E} \cdot \tilde{r} = 0$ so $\tilde{E} \perp \tilde{r}$ and \tilde{v}

phase velocity = propagation wave pattern
 wave $\tilde{r} = (k_x, k_y)$ equal phase: $k_x x + k_y y = \text{const}$ $x = (k_y/k_x) y + \text{const}$ $v_{\text{phase}} = \omega/k = c$

group velocity = propagation envelope non-monochromatic wave train $k_x \Delta k_x \approx 1/2$
 dispersion relation $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ and diagnostic diagram



shaded: real solutions = propagating waves
 boundary line: $k_x = k_x = 0$, waves only in x
 smaller k_x towards upper left
 $\omega = \text{avoid } k_x = 0$, waves only in y and z
 curve: $\omega = \sqrt{\omega_c^2 + c^2 k^2}$ for constant $k_x \neq 0$
 long periods: only at large wavelength $\lambda = 2\pi/k$
 non-shaded: period' too long for λ , wavelength λ , too small for ω_c
 imaginary solutions: $k_x \neq 0$ or $k_x = 0$: exponential growth/decay in y or z

6

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 2

reordered dispersion relations
 $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$

Burnt Väisälä frequency Schwarzschild convective instability criterion: $N^2 < 0$
 $\omega_c = \sqrt{\gamma - 1} N_0 = \sqrt{\gamma - 1} \frac{d \ln P_0}{d \ln \rho_0}$ isothermal $\omega_c = N_0$ $\gamma = 5/3$ $\omega_c = 0.98 \omega_c$

with $\tilde{P}_1 = \tilde{P}_1 e^{i(kx - \omega t)}$ and $\tilde{v}_1 = \tilde{v}_1 e^{i(kx - \omega t)}$
 $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$ $\tilde{\omega}^2 = \tilde{\omega}^2 + \tilde{k}^2 c^2$

diagnostic diagram
 shaded: two roots per (k_x, k_y) = propagative waves
 $\omega > \omega_c$ and $\omega > \omega_c$: acoustic wave
 $\omega < \omega_c$ and $\omega < \omega_c$: internal gravity wave $\omega \neq \omega_c$



blank: $k_x = 0$: evanescent in x

line $\omega = \omega_c$ ($k_x = 0$): horizontal "Lamb" waves
 line $\omega = \omega_c$ ($k_x \neq 0$): gravity-wave cutoff at small k_x (not shown)
 upper curve: constant k_x , hyperbolic, $\omega = \omega_c$ = ω_c for small k_x
 lower curve: constant k_x , hyperbolic, $\omega = \omega_c$ = ω_c for large k_x
 arrows: increasing k_x

9

10

SLANTED AG WAVES IN ISOTHERMAL ATMOSPHERE 3

Ste Fig 5.12

phase speed $\theta = \text{angle between } \vec{v} \text{ and } \vec{k}_z$

$$v_{\text{phase}}^2 = \frac{c^2}{1 - \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta} = c^2 \left(\frac{1}{1 - \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta} \right)$$

$v_{\text{phase}}^2 \approx c^2 \left(1 + \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta \right)$ for $v_{\text{flow}} \ll c$
 $v_{\text{phase}}^2 \approx c^2 \left(1 + \frac{v_{\text{flow}}^2}{c^2} \right)$ for $\theta = \pi/2$
 $v_{\text{phase}}^2 = c^2$ for $\theta = 0, \pi$

group velocity

$$\frac{d\omega}{dk} = \frac{c^2 k_z}{\omega} = \frac{c^2 \cos \theta}{v_{\text{phase}}}$$

pure p regime $\omega \gg \omega_{\text{ce}}$
 $\omega^2 \approx c^2 k^2$ $v_{\text{phase}} \approx c$ $v_{\text{group}} \approx c$ $v_{\text{phase}} \approx v_{\text{group}} \approx c$ P_1, P_2, P_3, P_4 in phase

pure s regime $\omega \ll \omega_{\text{ce}}$
 $\omega^2 \approx \frac{c^2 k^2}{1 - \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta}$ $v_{\text{phase}} \approx \frac{c}{\sqrt{1 - \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta}}$ $v_{\text{group}} \approx \frac{c \cos \theta}{\sqrt{1 - \frac{v_{\text{flow}}^2}{c^2} \cos^2 \theta}}$ P_1, P_2, P_3, P_4 in phase

dispersion: $\omega = \omega(k)$ $\frac{d\omega}{dk} = v_{\text{group}}$
 non-vertical only ("bow" concept needs horizontal info)
 $\frac{d\omega}{dk_x} = v_{\text{group},x}$ $\frac{d\omega}{dk_y} = v_{\text{group},y}$ $\frac{d\omega}{dk_z} = v_{\text{group},z}$ $\frac{d\omega}{dk} = v_{\text{group}}$
 slanted upward waves transport energy downward

evanescent regime
 k_x imaginary, k_z real, θ and ω 90 deg out of phase
 non-radial fundamental mode (θ) - ocean wind wave: $\omega = \sqrt{gk}$

11

WAVE TRAPPING

Stein & Leibacher

1974ARA...12...4075

temperature stratification

sound speed $c = \sqrt{\gamma P / \rho} = \sqrt{\gamma R T}$
 acoustic cutoff frequency $\omega_{ac} = \frac{1}{2H} \sqrt{\frac{2\gamma R T}{\gamma R T - 1}} \approx \frac{1}{2H} \sqrt{2\gamma R T}$
 horizontal acoustic wave has $k_{\perp} \omega_{ac} > \omega$
 gravity cutoff line: $\omega = \omega_g = \sqrt{g/k}$
 horizontal gravity wave frequency $\omega = \omega_g = \omega_{ac} = \omega_{ce}$

wave trapping
 reflection of increasing wave speed (downward bending)
 low-frequency waves become horizontal (mode 1)
 and non-reflection in cutoff (first phase speed)

p-mode cavity walls: ω_{ac} and Lamb-line reflection
 p-mode cavity walls: gravity cutoff line and ω_{ce} reflection

example: three temperature regimes shaded: 2 = cavity p and s modes

12

SOLAR CAVITIES

Ste Fig 5.13

μ : refractive turnaround at Lamb line
 $\omega = \omega_{ac} = \frac{1}{2H} \sqrt{2\gamma R T}$
 lowest f modes probe deepest

μ : refractive turnaround at Brunt-Väisälä N^2
 no μ waves in convection zone
 interior p modes? In waves envelope? Atmospheric μ waves?

near solar center
 Lamb reflection $\omega^2 = 1/\rho^2$
 ω_{ac} and $\omega_{ce} \approx g \approx \omega$, $N^2 \approx \omega^2$
 only radial μ -l modes reach center

five-minute oscillation ($\omega = 0.1$ Hz)
 upper reflection $\omega = \omega_{ce}$ (near N_{max})
 interior reflection = Lamb turnaround
 (= closed/open ends of my flute)
 μ modes evanescent where detected

three-minute oscillation $\omega = 0.05$ Hz
 here: clearly from coronal F rise
 but: F rise too warped
 reflection-mode conversion at β

13

NUMERICAL μ -MODE PREDICTION

Ando & Osaki 1979PASJ...27...581A

(a) Basic Equations and Boundary Conditions

The basic equations governing nonadiabatic radial pulsations in the radiative atmosphere were formulated by Unno (1960), in which the radiative transfer was treated in the Eddington approximation. Unno and Spiegel (1966) have demonstrated that the Eddington approximation in the radiative heat equation is very useful in three-dimensional time-dependent problems. We thus utilize this in the present paper. The basic equations governing nonadiabatic nonradial oscillations in the radiative atmosphere are then given as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho} \nabla p - \mathbf{g}, \quad (2)$$

$$\rho \mu \left(\frac{dT}{dt} - \frac{\nabla \cdot \mathbf{F}}{\rho} + \frac{D}{dt} \right) = -\nabla \cdot \mathbf{F}, \quad (3)$$

$$\mathbf{F} = -\frac{4\pi}{3\kappa} \nabla T, \quad (4)$$

and

$$J = \frac{4\pi}{3} \rho \mu \frac{D}{dt} \left(\frac{dT}{dt} - \frac{\nabla \cdot \mathbf{F}}{\rho} + \frac{D}{dt} \right), \quad (5)$$

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THE SUN IS A SPHERE

Wolf 1972ApJ...177L...87W

Fig. 1—Radial distribution of kinetic energy (velocity units) in spherical shells of unit thickness in five oscillation periods of solar oscillation.

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NUMERICAL μ -MODE PREDICTION

Ando & Osaki 1979PASJ...27...581A

We assume as usual that the small perturbation of a physical quantity f is written as

$$\begin{aligned} f'(r, \theta, \phi, t) &= \frac{f'(r)}{Y_l^m(\theta)} Y_l^m(\phi) e^{i\omega t} \\ \frac{df'(r, \theta, \phi, t)}{dt} &= \frac{df'(r)}{dt} \end{aligned} \quad (6)$$

where (r, θ, ϕ) is the spherical polar coordinates, the Eulerian and the Lagrangian perturbations are denoted by prime ($'$) and δ , and $Y_l^m(\theta, \phi)$ is the spherical harmonics. We introduce a nondimensional frequency ω and five nondimensional variables $\alpha, \beta, \theta, \phi, j$, and z defined by

$$\begin{aligned} \omega &= (R/gM)^{1/2} \omega', \\ \beta &= \beta r / R, \quad \theta = \theta' / T, \\ j &= j' / T, \quad \phi = \phi' / L, \\ z &= z' / R, \end{aligned} \quad (7)$$

where $\omega = \omega' (R/gM)^{1/2}$ is the luminosity at radius r and L is the luminosity at the surface. By linearizing equations (1)-(5), we then obtain four first-order linear differential equations and one auxiliary equation:

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POWER RIDGES $f - 1$ (aka $k - 1$) DIAGRAM

Ste Fig 5.2

assume constant temperature gradient with depth $d = z$
 $\frac{dT}{dz} = C = \left(\frac{dT}{dz} \right)_0$ $T(d) = \left(\frac{dT}{dz} \right)_0 d = \frac{1}{2} \left(\frac{dT}{dz} \right)_0 \frac{(1-1)^2}{(1-1)^2} = \frac{1}{2} \left(\frac{dT}{dz} \right)_0 (1-1)^2$

lower reflection occurs at depth $d = d$ where $\omega^2 = c^2 k^2$ $d = \frac{c^2}{2 \left(\frac{dT}{dz} \right)_0} \frac{\omega^2}{(1-1)^2} = \frac{c^2}{2 \left(\frac{dT}{dz} \right)_0} \frac{\omega^2}{(1-1)^2}$

standing wave has $(\alpha + 1/2) \pi = \int_0^d k_z dz = \int_0^d \frac{\omega}{\sqrt{c^2 - 2 \left(\frac{dT}{dz} \right)_0 z}} dz = \frac{2\omega \sqrt{c^2}}{\sqrt{c^2 - 2 \left(\frac{dT}{dz} \right)_0}} \frac{1}{(1-1)^2} \pi$

parabolas $\omega^2 = \frac{1}{2} \left(\frac{dT}{dz} \right)_0 (1-1)^2 (1-1)^2 k_z^2$ $k_z^2 = (1 + 1/2) \frac{2\omega^2}{c^2}$ Ste Fig 5.1f

Duvall's law with $\alpha = 1.58$

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FIVE-MINUTE OSCILLATION THEORY

Robert J. Rutten
<http://www.staff.science.uu.nl/~rutte101>

Literature
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Ste Fig 5.12

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