Toy black holes and entropy quantization in LQG

Hanno Sahlmann

Spinoza Institute, Utrecht University

(See also arXiv: 0709.0076,0709.2433)

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These topics are indirectly related.

0. Black Holes in LQG – a Reminder

Long and beautiful story (Rovelli, Ashtekar, Baez, Corichi, Krasnov, ...).

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- quasi-local notion of black hole horizon: **Isolated horizon**
- quantize classical theory containing such a horizon as **inner boundary of space-time**

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$\Longrightarrow$
- KH is IH,
- IH inside/coincides with EH,
- IH + assumptions $\Rightarrow \exists$ EH.

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$\Longrightarrow$ U(1)-Chern Simons boundary term in action/symplectic structure.
The boundary conditions in canonical variables (A, E)

$$-\frac{2\pi\gamma}{a_0} E_I r^I|_\Delta = dW (\equiv d(A^I r_I)|_\Delta)$$

Quantum theory:

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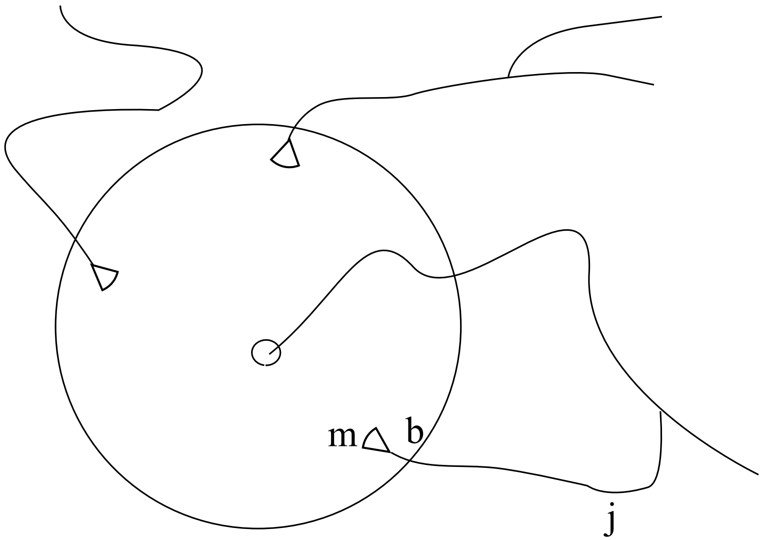
BH horizon punctured by spin-network edges:



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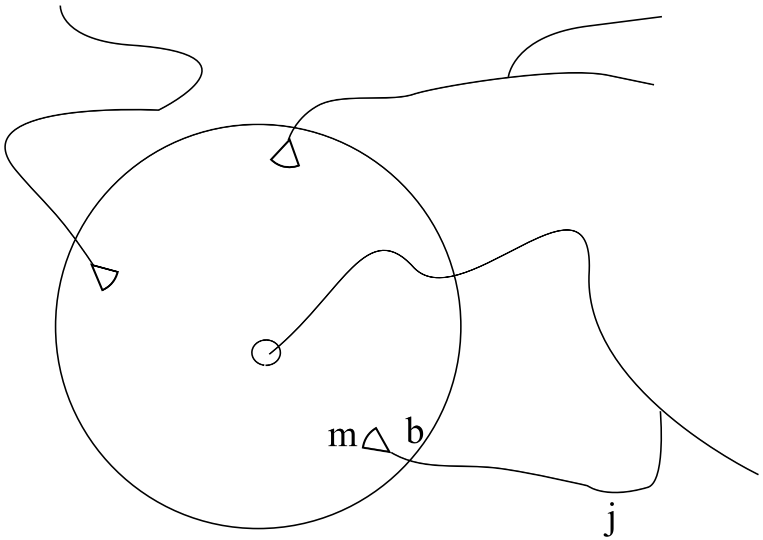

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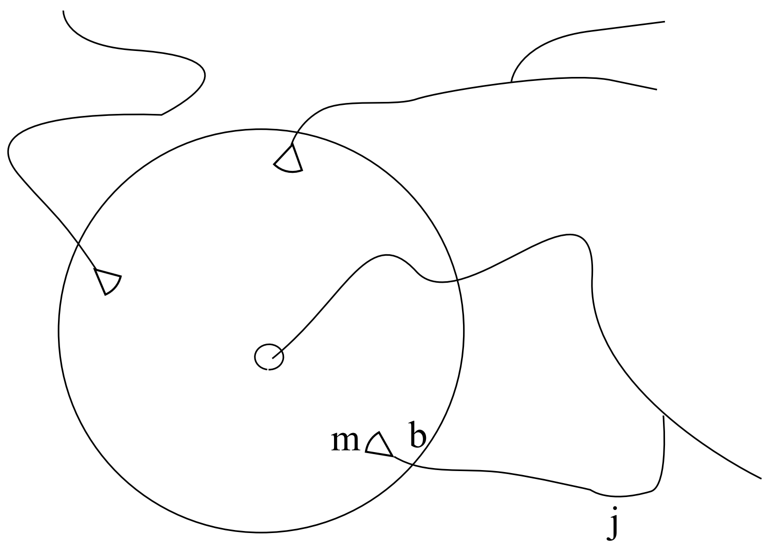

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Bulk states: $|(j_1, m_1; j_2, m_2; \ldots)(\text{ more })\rangle$. $j_i \in \mathbb{N}_*/2$, $m_i \in \{-j, -j+1, \ldots, j\}$.

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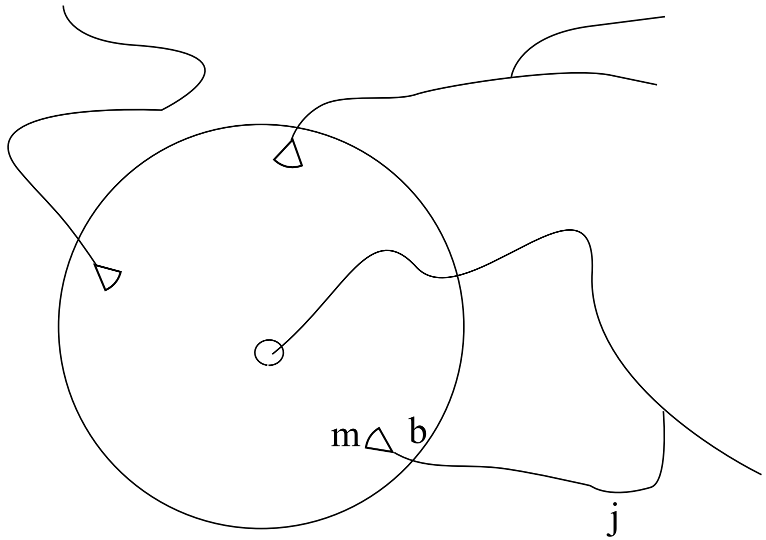

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j-labels $\longleftrightarrow$ area: $A_j = 8\pi\gamma l_P^2 \sqrt{j(j+1)}$

Physical states:

$$(\text{surface states}) \bigotimes (\text{bulk states}) \text{ subject to}$$

$$2m_i = b_i \mod k \quad \text{and} \quad \sum b_i = 0$$

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Lewandowski and Domagala:

$$N(a) = \left| \left\{ (m_1, m_2, \ldots),\ m_i \in \mathbb{Z}_*/2 : \sum_i m_i = 0, \sum_i \sqrt{|m_i|(|m_i|+1)} \approx a \right\} \right|$$

Results so far (Meissner, L&D):

$$S(A) = \ln N(A/8\pi\gamma l_P^2) = \frac{\gamma_M}{\gamma}\frac{A}{4l_P^2} - \frac{1}{2}\ln\left(\frac{A}{l_P^2}\right) + O(A^0)$$

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Motivation for what will follow:

- ✘ some sweeping approximations go into above result. $\longrightarrow$ Check!
- ✘ Combinatorics hard. What if have crazy hypothesis, or want to change something and see?
- ✘ All just asymptotics. There is surprising "microstructure" in BH area spectrum.

1. The Toy Black Hole

We can expand

$$\sqrt{j(j+1)} = j + \frac{1}{2} - \frac{1}{4(2j+1)} - \frac{1}{16(2j+1)^3} + \ldots$$

so let us do fantasy LQG:

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$$A_j = 8\pi\gamma l_P^2 \sqrt{j(j+1)} \qquad A_j \doteq 8\pi\gamma l_P^2 \left(j + \frac{1}{2}\right)$$

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This simplifies things tremendously. (And maybe not completely fantasy.)

Turns with new spectrum number of states is

$$N(a) \doteq \left| \left\{ (m_1, m_2, \ldots),\ m_i \in \mathbb{Z}_* \ :\ \sum_i m_i = 0, \sum_i (|m_i| + 1) = a \right\} \right|.$$

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Will actually look at slightly more general problem:

$$N(a, j) \doteq \left| \left\{ (m_1, m_2, \ldots),\ m_i \in \mathbb{Z}_* \ : \ \sum_i m_i = j, \sum_i (|m_i| + 1) = a \right\} \right|.$$

First few values of $N(a, j)$:

0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	2	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	2	0	2	0	2	0	2	1	0	0	0	0	0	0
0	0	0	0	0	1	3	1	2	3	2	3	2	1	3	1	0	0	0	0	0
0	0	0	0	1	4	3	2	6	2	6	2	6	2	3	4	1	0	0	0	0
0	0	0	1	5	6	3	9	6	9	8	9	6	9	3	6	5	1	0	0	0
0	0	1	6	10	6	12	14	12	18	12	18	12	14	12	6	10	6	1	0	0
0	1	7	15	12	16	26	20	32	25	34	25	32	20	26	16	12	15	7	1	0
1	8	21	22	23	42	38	50	53	54	58	54	53	50	38	42	23	22	21	8	1

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0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	2	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	2	0	2	0	2	0	2	1	0	0	0	0	0	0
0	0	0	0	0	1	3	1	2	3	2	3	2	1	3	1	0	0	0	0	0
0	0	0	0	1	4	3	2	6	2	6	2	6	2	3	4	1	0	0	0	0
0	0	0	1	5	6	3	9	6	9	8	9	6	9	3	6	5	1	0	0	0
0	0	1	6	10	6	12	14	12	18	12	18	12	14	12	6	10	6	1	0	0
0	1	7	15	12	16	26	20	32	25	34	25	32	20	26	16	12	15	7	1	0
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Can we get an explicit formula?

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0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	0	2	0	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	2	0	2	0	2	0	2	1	0	0	0	0	0	0
0	0	0	0	0	1	3	1	2	3	2	3	2	1	3	1	0	0	0	0	0
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We can get the generating function.

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$$G(g,z) \doteq \sum_{a=0}^{\infty} \sum_{j=-a}^{a} N(a,j) g^a z^j.$$

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$$G(g,z) = \sum_{m=1}^{\infty} (G_1(g,z))^m = \frac{g^2 \left(z^2 - 2gz + 1\right)}{(g+1)\left(2zg^2 - \left(z^2 + z + 1\right) g + z\right)}.$$

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$$G_{\leq}^{(j=0)}(g) \doteq \sum_{a=0}^{\infty}\left(\sum_{b=0}^{a} N(b,0)\right) g^a = \frac{1}{1-g} G^{(j=0)}(g)$$

Asymptotics

Heuristics: If

$$f(x) = \sum_{n=0}^{\infty} c_n x^n \qquad \text{and} \qquad R = |\text{Pole of } f(x) \text{ closest to } 0|$$

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Expect $N(a,0) \propto 2^a$.

Theorems show

$$N(a,0) \sim \frac{1}{6\sqrt{\pi}} \frac{2^a}{\sqrt{a}}, \qquad \sum_{b}^{a} N(b,0) \sim \frac{1}{3\sqrt{\pi}} \frac{2^a}{\sqrt{a}}$$

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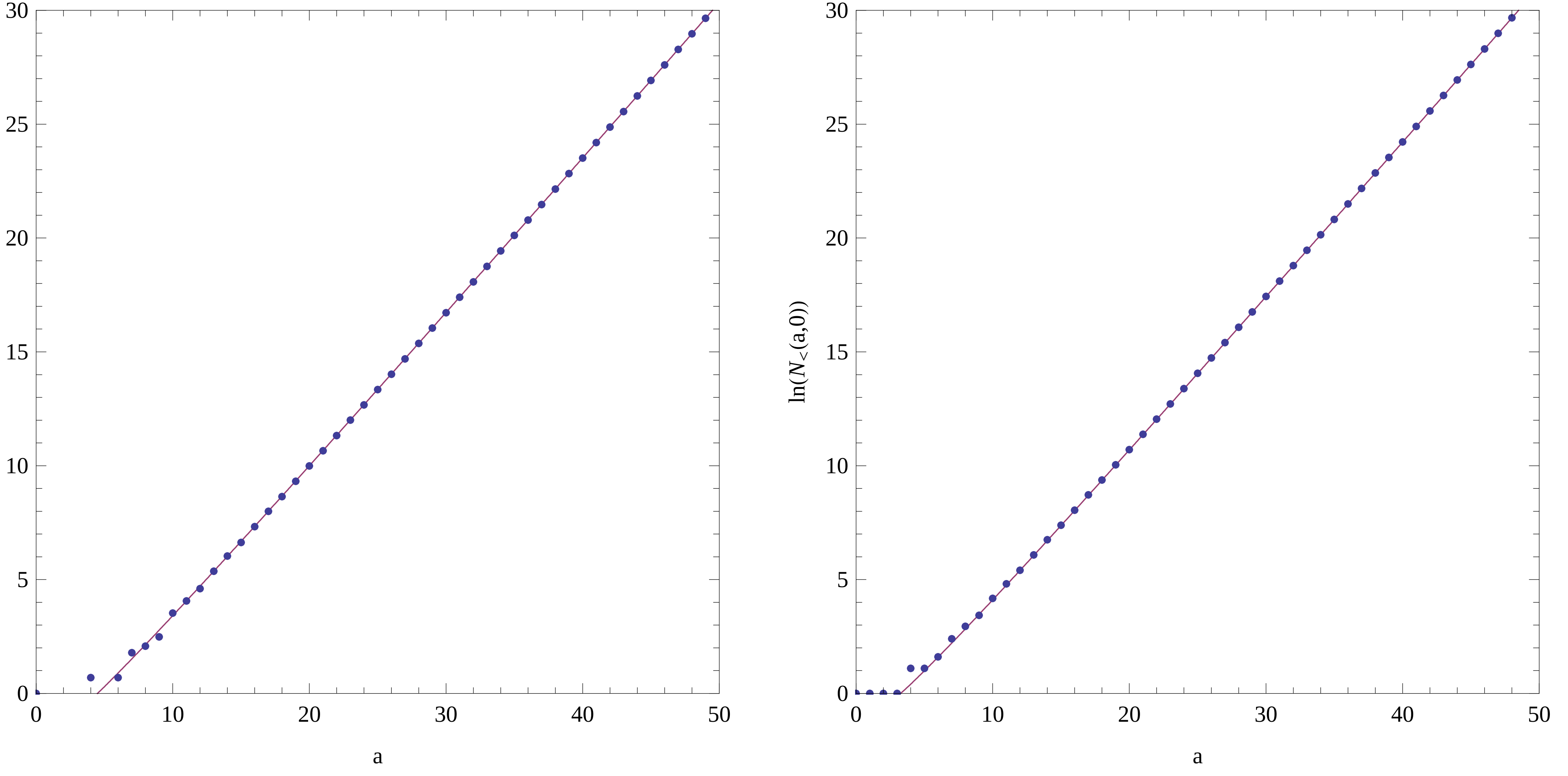
$$N(a,0) \sim \frac{1}{6\sqrt{\pi}}\frac{2^a}{\sqrt{a}}, \qquad \sum_{b}^{a} N(b,0) \sim \frac{1}{3\sqrt{\pi}}\frac{2^a}{\sqrt{a}}$$

and we already saw that

$$\sum_{j} N(a,j) \sim 2^a.$$



0
5
10
15
20
25
30
0
10
20
30
40
50
a
ln($N_<$(a,0))
a

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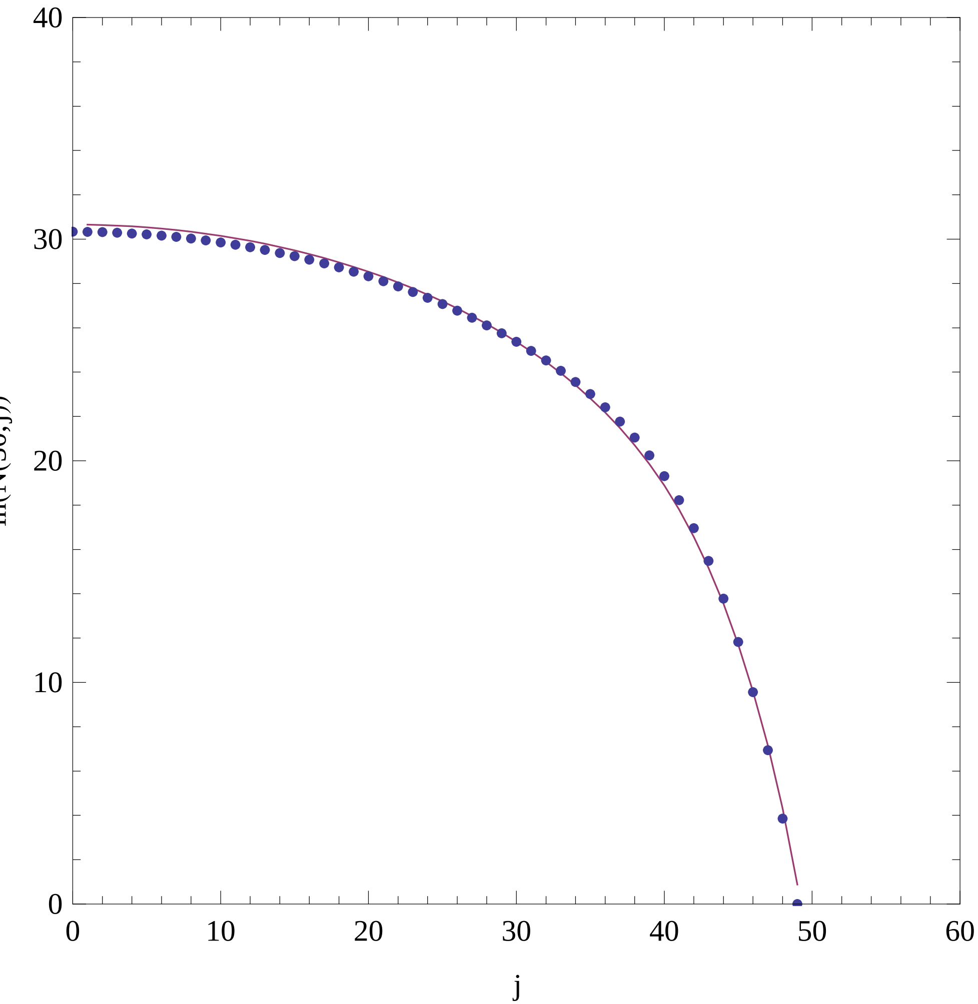

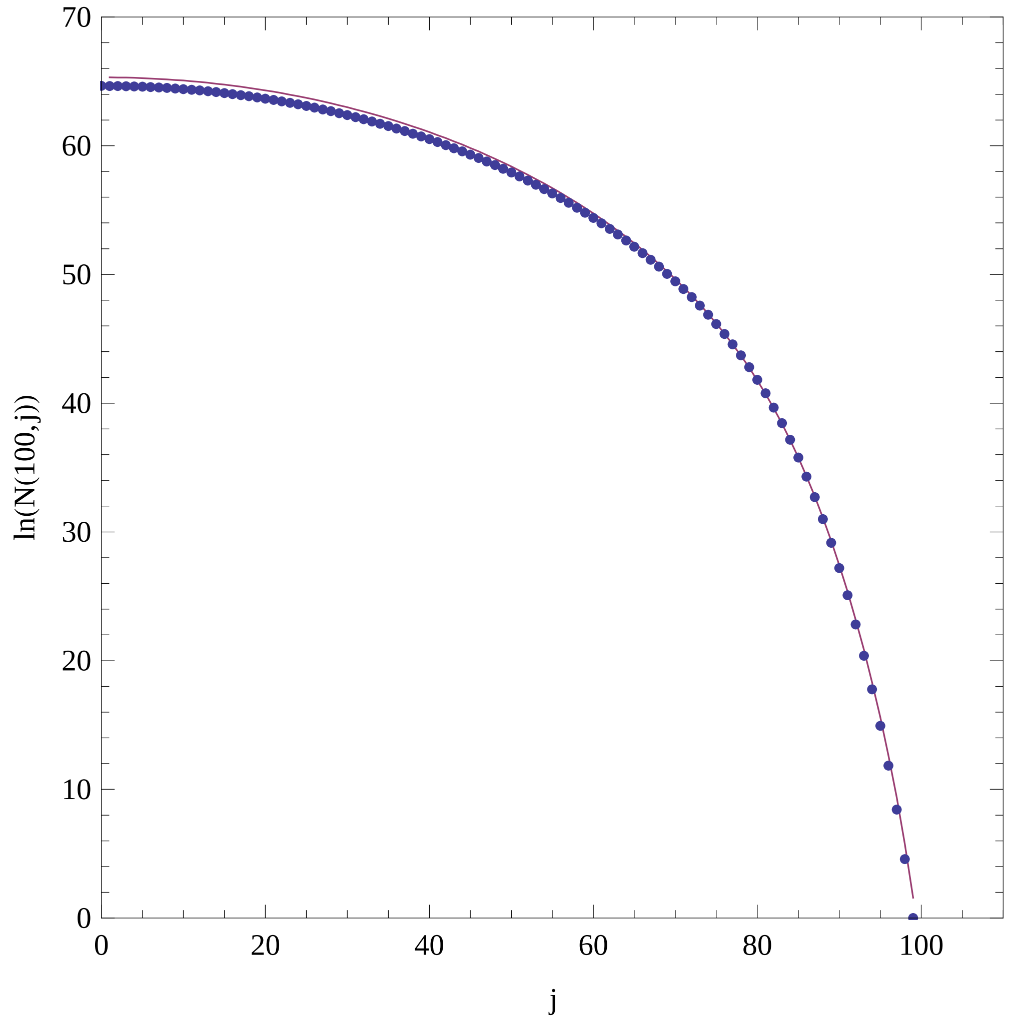

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Altogether: Nice toy model

2. Entropy Quantization

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- Dreyer: Can get $\Delta A = 4\ln(3)$ as A_1 and gauge group SO(3)
- Domagala, Meissner, Lewandowski: Actually, no.

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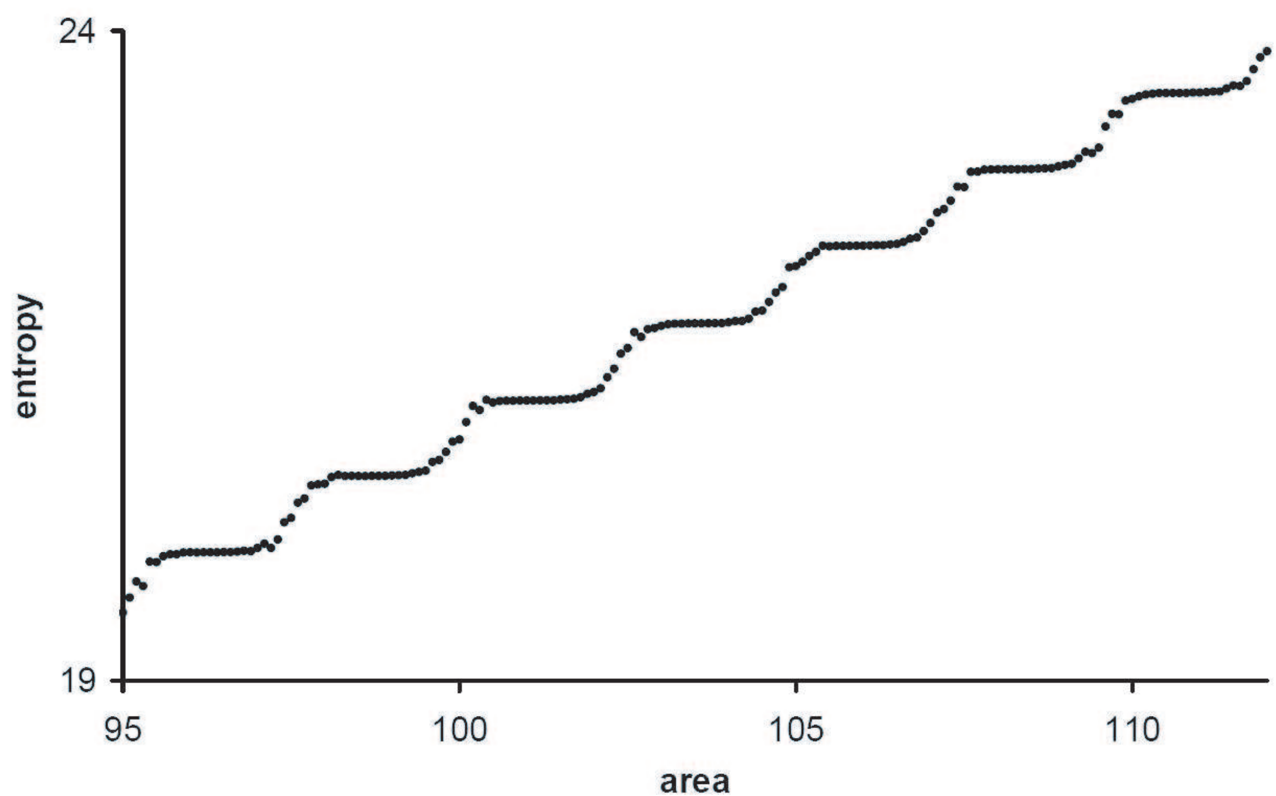

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Furthermore: Phenomenon contingent on implementing quantum boundary conditions

Idea

1. Look at the problem in terms of $N(I, j)$:

$$N(I, j) = \left| \left\{ (m_1, m_2, \ldots),\ m_i \in \mathbb{Z}_*/2 : \sum_i m_i = j, \sum_i \sqrt{|m_i|(|m_i| + 1)} \in I \right\} \right|$$

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2. View state labels as paths in a certain space

3. Use statistics of steps in these paths to explain pattern.

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Physical states $\longleftrightarrow$ paths that end on $\mathbb{R}_+ \times \{0\}$

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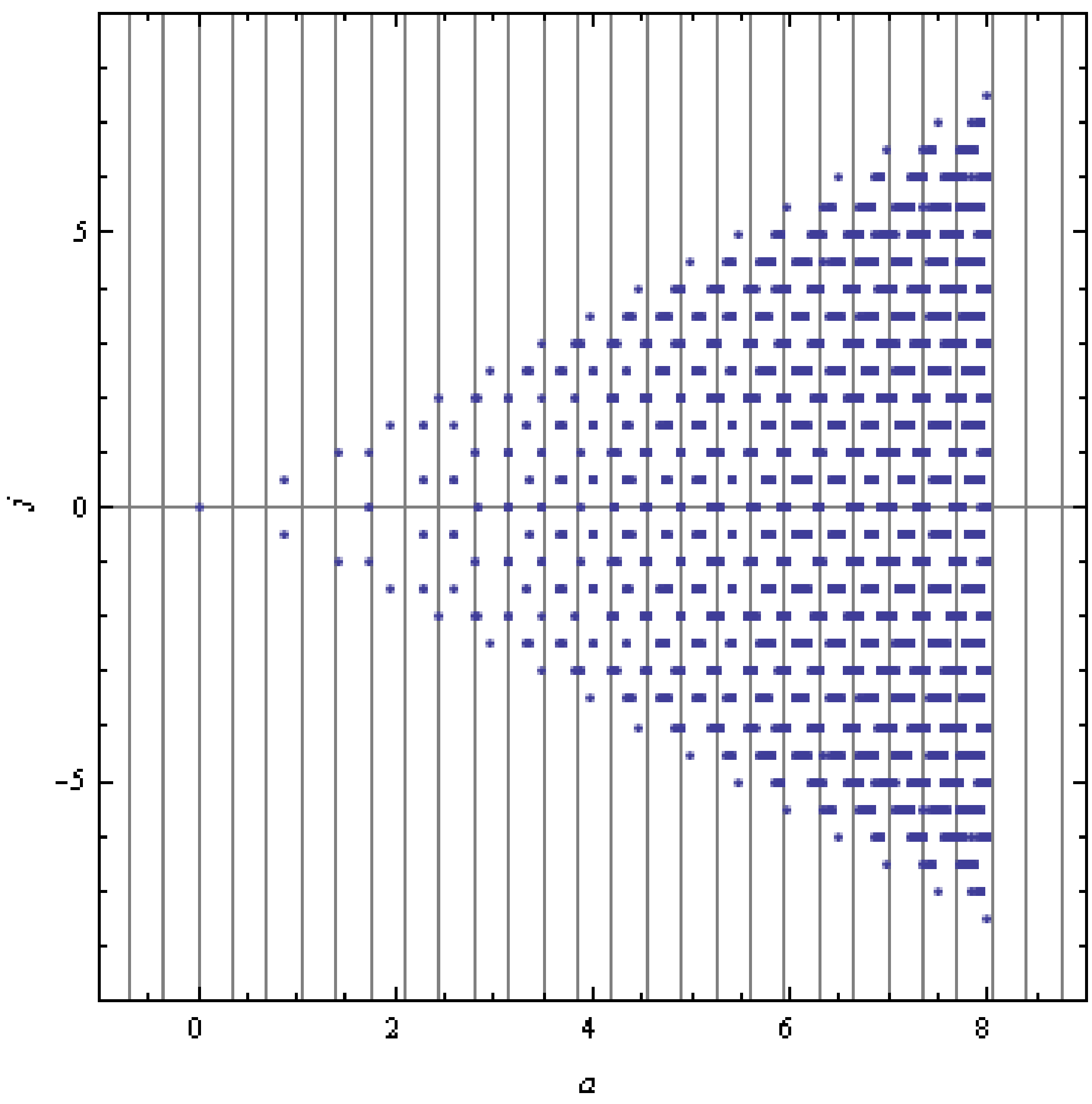

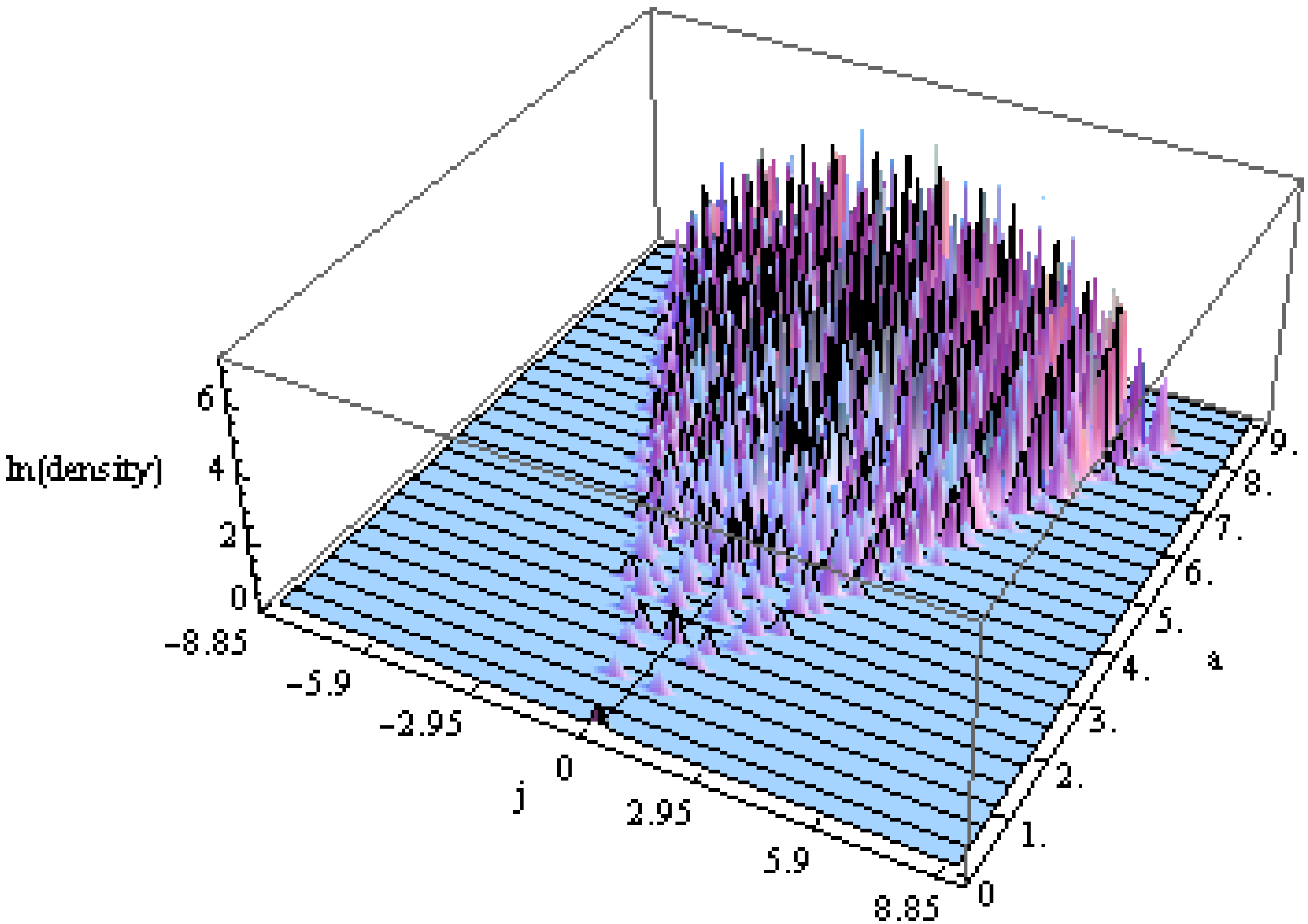

ln(density)
6
4
2
0
-8.85
-5.9
-2.95
0
2.95
5.9
8.85
j
0
1.
2.
3.
4.
5.
6.
7.
8.
9.
a

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- use statistics!
- maybe we have $s(m) = I(m)s_0$, with $I(m) \in \mathbb{N}$ *on average*?

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Then the cental limit theorem says

$$\langle \delta(n) \rangle \approx n \langle \epsilon(m) \rangle, \qquad \langle \delta(n)^2 - \langle \delta(n) \rangle^2 \rangle \approx n \langle \epsilon(m)^2 - \langle \epsilon(m) \rangle^2 \rangle,$$

This furnishes explanation of the clustering if

- ✘ $\langle \epsilon(m) \rangle = 0$
- ✘ small variance:

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How does this work out in practice? Will need

- information about probability distributions
- educated guess for $I(m)$

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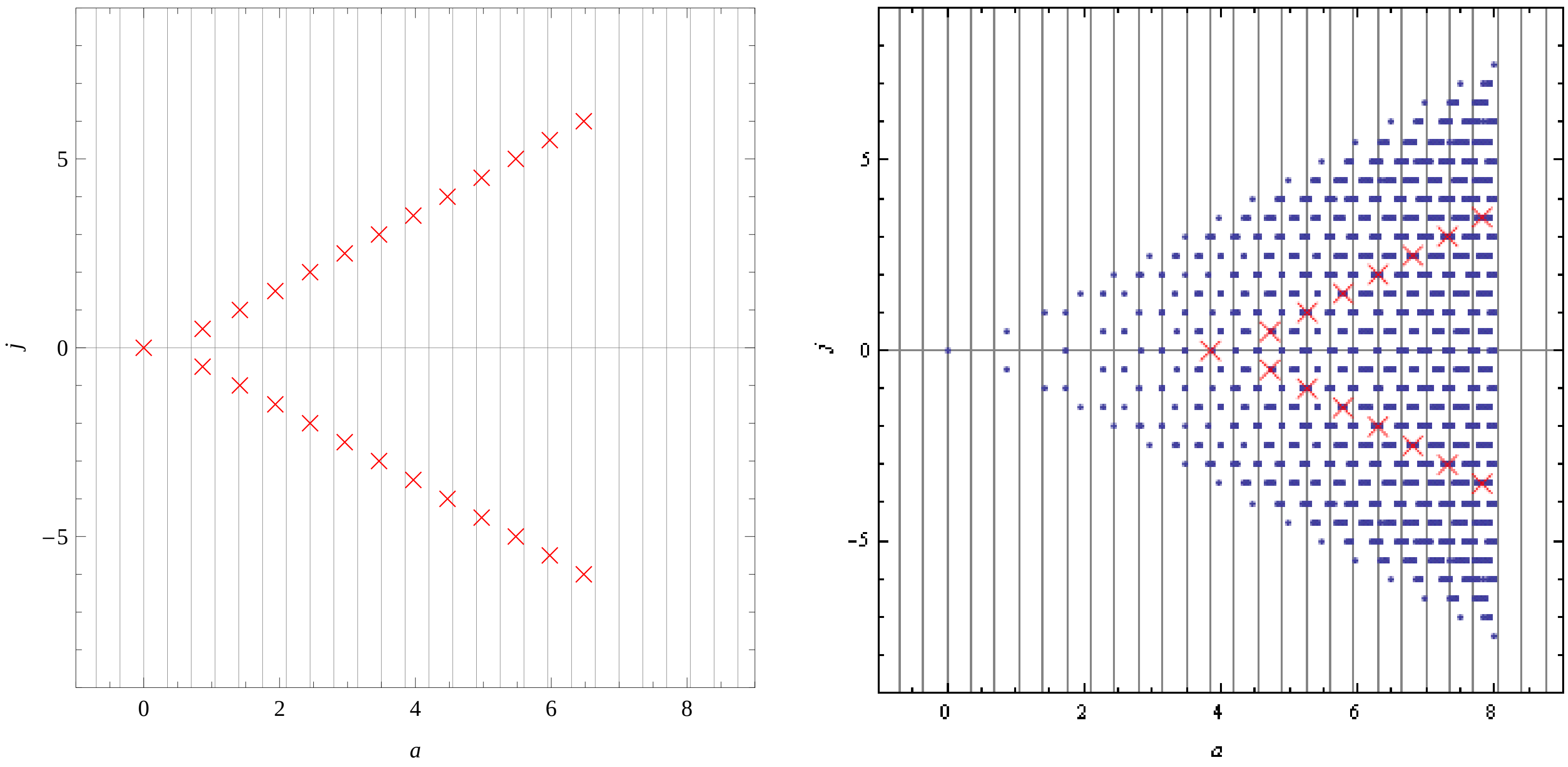
Determining $I(m)$



$$a(m) = \left(\frac{3}{2} \cdot 2m + 1\right) \Delta a + \epsilon(m).$$

Results

The requirement $\langle \epsilon(m) \rangle = 0$ implies

$$\Delta a = \frac{\langle a(m) \rangle}{3\langle m \rangle + 1} \quad \text{and} \quad \epsilon(m) = a(m) - (3m + 1)\frac{\langle a(m) \rangle}{3\langle m \rangle + 1}.$$

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This can be evaluated numerically. We find

$$\Delta a \approx 0.34952, \qquad \langle \epsilon(m)^2 \rangle \approx 0.00019156$$

What does that mean?

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- Result for Δa compares nicely with CDF:

$$\chi \approx 8.7843, \qquad \chi_{\mathrm{CDF}} \approx 8.80 \qquad \frac{\chi_{\mathrm{CDF}} - \chi}{\chi_{\mathrm{CDF}}} \approx 0.00129$$

- We seem to be even closer to the conjectured value:

$$8\ln(3) \approx 8.7889, \qquad \frac{8\ln(3) - \chi}{\chi} \approx 0.00053.$$

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Altogether: Mixed bag. Should try analytic approach.